THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV22 CONFERENCE

1 & 2 DECEMBER

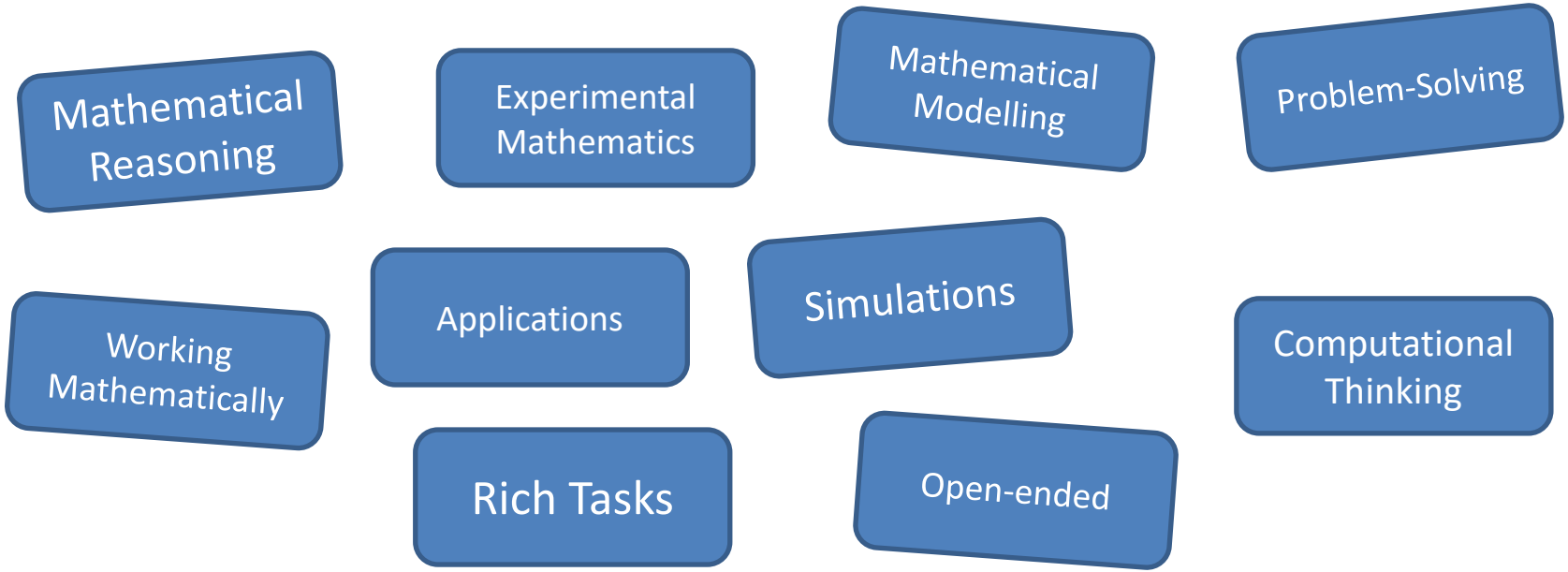
VALUING MATHEMATICS IN A CHANGING WORLD

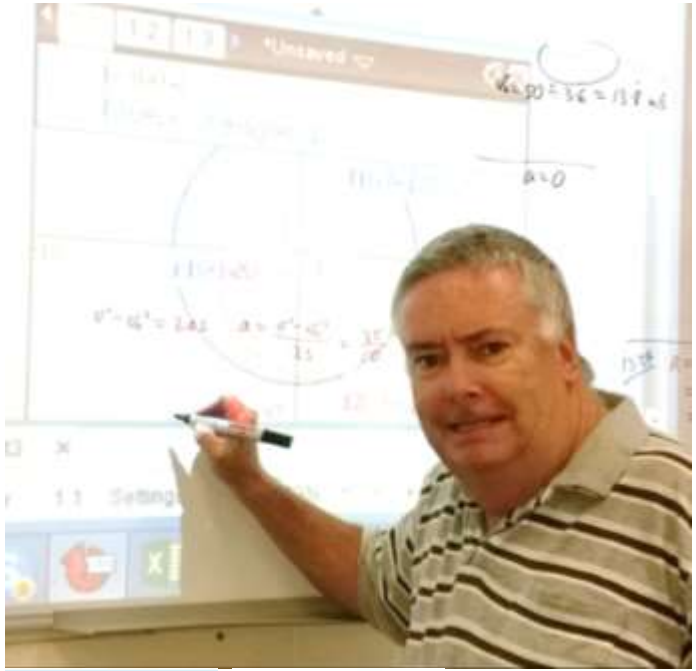
COMPUTATIONAL THINKING IN PROBLEM-SOLVING, AND ITS APPLICATION TO MATHEMATICAL INVESTIGATION

Brian Lannen

Warm-up activity – turn & talk

What do you think defines a mathematical investigation? Is it any different from a project?





Brian Lannen has been teaching for over 35 years in Victoria and NSW. He has taught Physics, Maths and Science in schools, university and TAFE colleges, was a curriculum consultant in NSW and New York and has contributed to a range of text-book writing projects. He helped establish T-Cubed (Teachers Teaching with Technology) in Australia in the 1990s and is now a Senior Mentor to that association and Principal Host of the Texas Instruments Australia webinar program.

T³

MURRAY MATHEMATICS

E14 COMPUTATIONAL THINKING IN PROBLEM-SOLVING, AND ITS APPLICATION TO MATHEMATICAL INVESTIGATION

(Technology to enhance investigation, Exploring effective pedagogies)

Brian Lannen, Murray Mathematics Curriculum Services (Year 9 to Year 12)

How much do you think about thinking? Algorithmic processes are a natural part of mathematics and, although we can assign some of this to technology, humans need to also play an active and creative role in the partnership. No wonder there is such a focus on this in the newly revised VCE Study Design. In this workshop we explore the notion of what this thinking is all about and examine the principles of setting up simulations and processes to model and investigate problems. The discussion will be illuminated with examples suited to problem solving and investigations from Years 9 to 12.

Plan for this session

- Definitions & Main Ideas
  - Computational Thinking
  - Problem-Solving
  - Mathematical Investigation
- VCE Study Design
- **Example Activities**
- More Resources

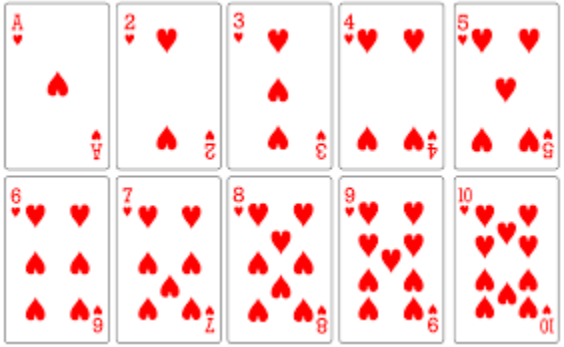
Algorithmic Thinking

- Algorithmic processes are a natural part of mathematics.
- Algorithmic Thinking is the kind of ‘thinking’ that a computer could execute.
- A human needs to be creative in algorithmic thinking, whereas a computer is mindlessly mechanical in its execution of a coded algorithm.
- Algorithm: Well defined set of sequential instructions designed to perform a particular task or solve a problem.
- Coding: A set of instructions (particular to a defined language or set of rules) by which algorithms are represented and implemented.

Key Points

- The focus is on helping students develop their thinking, NOT on having them memorise computational algorithms.
- Algorithmic Thinking is NOT the same as Coding.

A challenge for you:



You have a set of cards numbered 1 to 10.
Shuffle the cards into a random order.
Now design an algorithmic process by which you could instruct another person (or a robot) to place the cards in order from lowest to highest.
Is your algorithm absolutely foolproof?
Could your algorithm be represented on a flowchart or coded for a computer to execute?

Victorian Certificate of Education

Mathematics

Study Design

Accreditation Period
2023–2027

- apply computational thinking and algorithms, and use technology effectively as a tool for working mathematically.

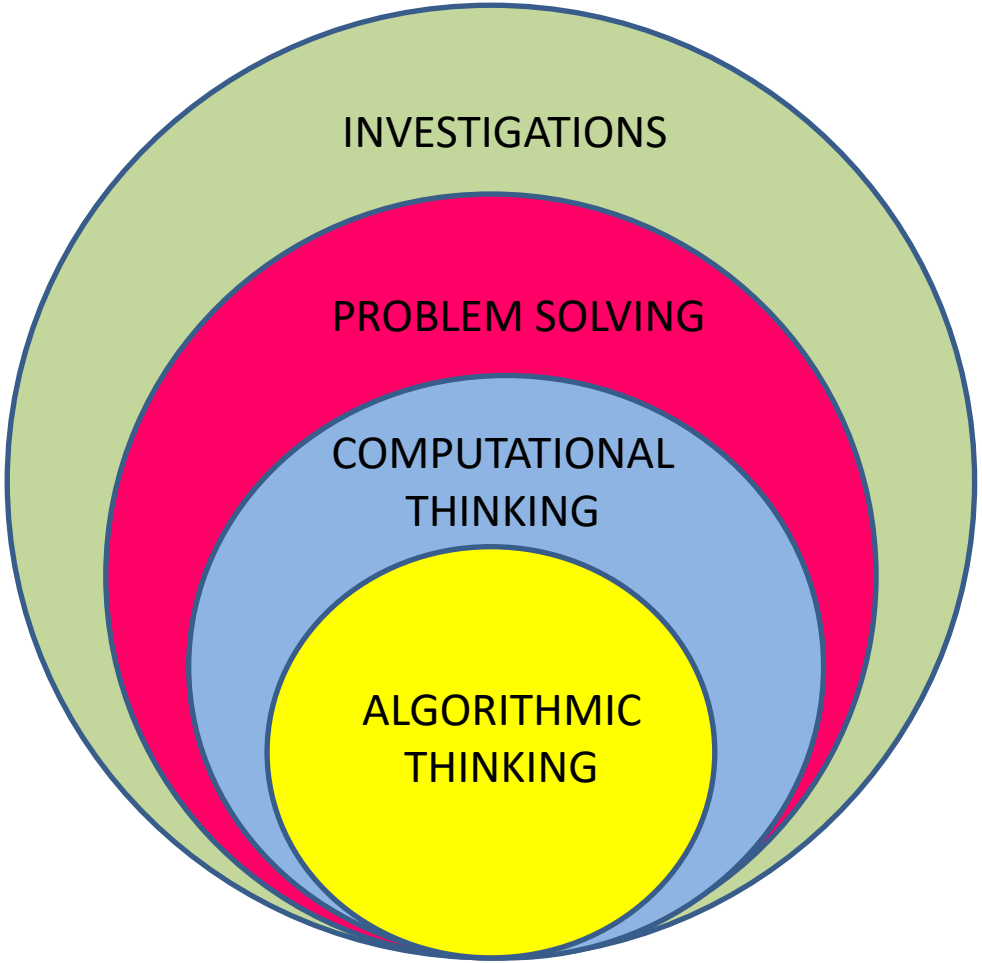
  Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Key knowledge

- the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation

Key skills

- use computational thinking, algorithms, models and simulations to solve problems related to a given context


INVESTIGATIONS
PROBLEM SOLVING
COMPUTATIONAL THINKING
ALGORITHMIC THINKING

Some definitions of Computational Thinking

Australian CURRICULUM

Download content

F-10 Curriculum | Senior secondary curriculum | Parent information | Student diversity | Resources

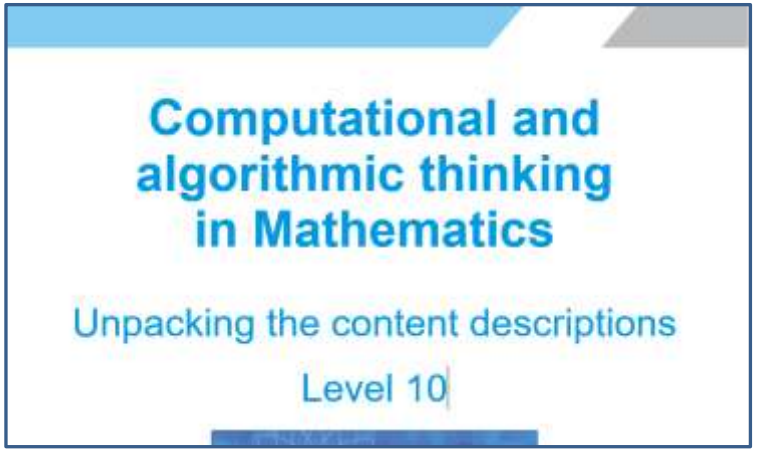
Search by keyword or content

Computational thinking

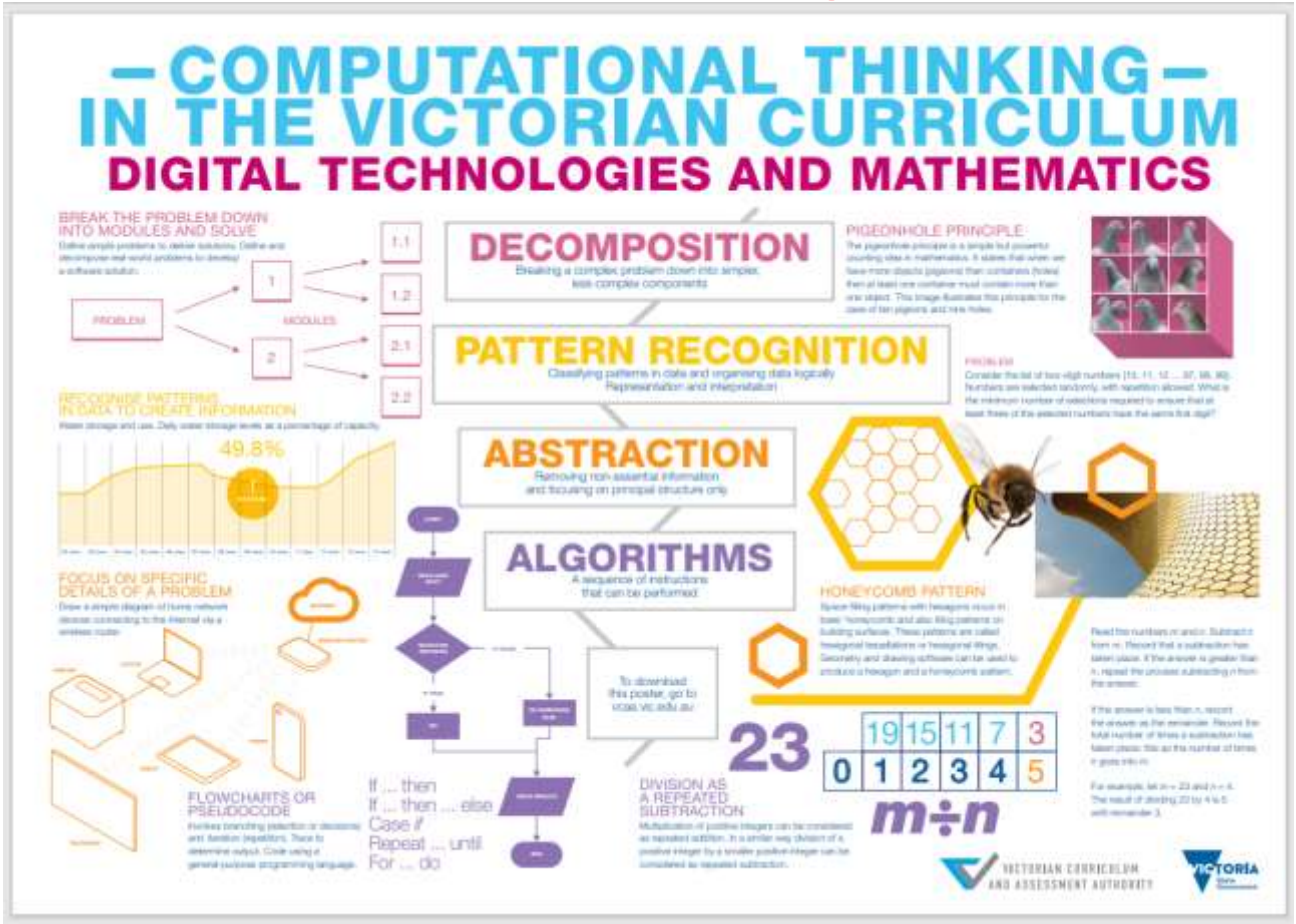
Computational thinking is a problem-solving method that is applied to create solutions that can be implemented using digital technologies. It involves integrating strategies, such as organising data logically, breaking down problems into parts, interpreting patterns and models and designing and implementing algorithms.

Computational thinking is used when specifying and implementing algorithmic solutions to problems in Digital Technologies. For a computer to be able to process data through a series of logical and ordered steps, students must be able to take an abstract idea and break it down into defined, simple tasks that produce an outcome. This may include analysing trends in data, responding to user input under certain preconditions or predicting the outcome of a simulation.

This type of thinking is used in Design and Technologies during different phases of a design process when computation is needed to quantify data and solve problems. Examples include when calculating costs, testing materials and components, comparing performance or modelling trends.



Some definitions of Computational Thinking



Some definitions of Problem-Solving

Google

problem solving

All Images Videos News Books More Settings Tools

About 305,000,000 results (0.54 seconds)

Problem solving is the act of defining a **problem**; determining the cause of the **problem**; identifying, prioritizing, and selecting alternatives for a solution; and implementing a solution. The **problem-solving** process. **Problem solving** resources.

asq.org › quality-resources › problem-solving

What is Problem Solving? Steps, Process & Techniques | ASQ

About Featured Snippets Feedback

Some definitions of Problem-Solving

Collins

English: problem-solving | Example sentences | Tren

Definition of 'problem-solving'

problem-solving

in British English

(ˈprɒbləmsɒlvɪŋ)

NOUN

the act or process of finding solutions to problems, esp by using a scientific or analytical approach

Problem-solving is often carried on by processes of visualization.
an approach to problem-solving

Derived forms

problem-solver NOUN

Oxford Reference

OVERVIEW

problem solving

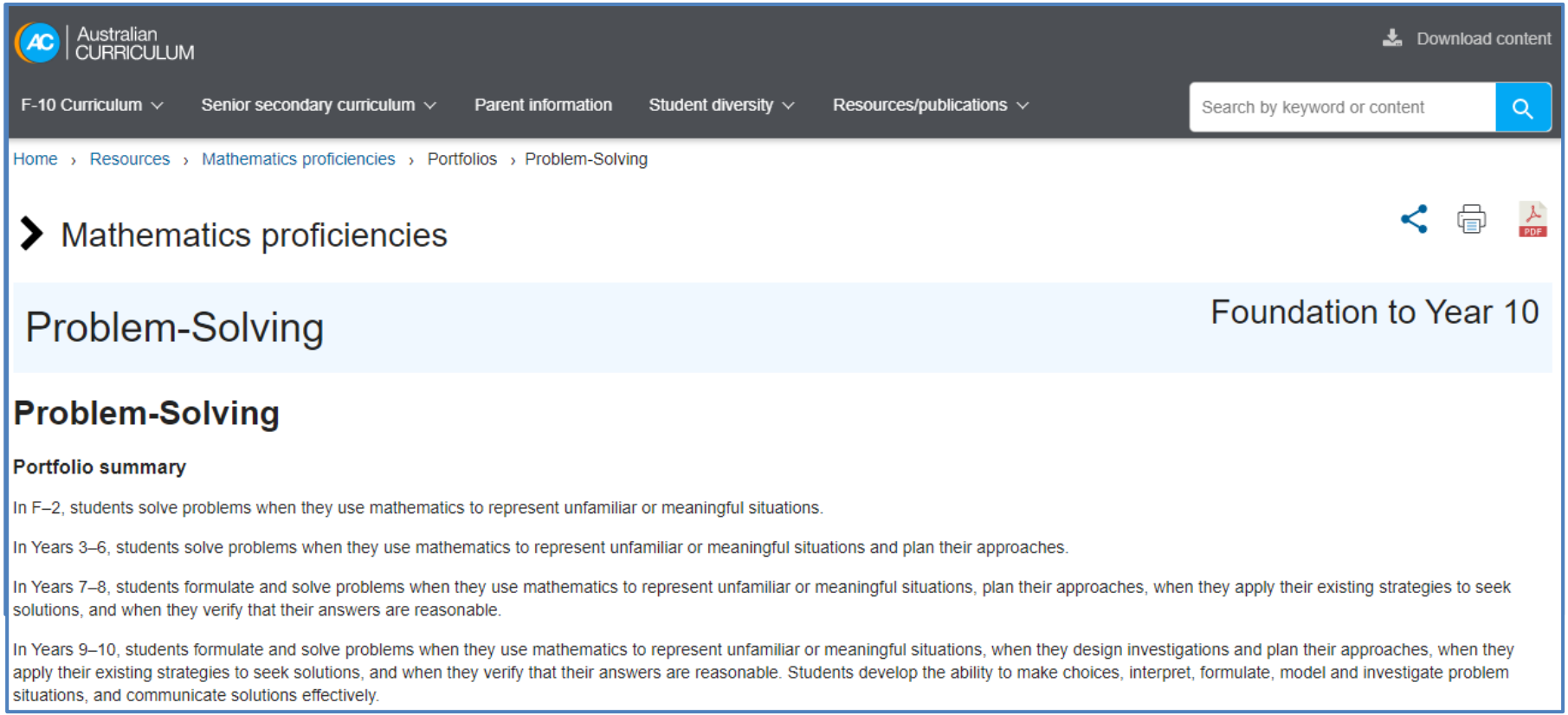
QUICK REFERENCE

Cognitive processing directed at finding solutions to well-defined problems, such as the Tower of Hanoi, Wason selection task, or a water-jar problem, by performing a sequence of operations. Problem solving by means of logic or logical analysis is usually called reasoning. See also 2-4-6 problem, algorithm, brute force algorithm, convergence-divergence, functional fixedness, General Problem Solver, ill-defined problem, insight (2), intelligence, lateral thinking, Monty Hall problem, muddy children problem, Newcomb's problem, nine-dot problem, oddity problem, problem-solving stages, taxicab problem, travelling salesman problem, well-defined problem. [From Latin *problema*, something put forward, from Greek *pro* before or forward + *ballein* to throw]

From: problem solving in A Dictionary of Psychology »

Subjects: Science and technology — Psychology

Some definitions of Problem-Solving



Australian CURRICULUM

Download content

F-10 Curriculum | Senior secondary curriculum | Parent information | Student diversity | Resources/publications

Search by keyword or content

Home › Resources › Mathematics proficiencies › Portfolios › Problem-Solving

Mathematics proficiencies

Problem-Solving

Foundation to Year 10

Problem-Solving

Portfolio summary

In F–2, students solve problems when they use mathematics to represent unfamiliar or meaningful situations.

In Years 3–6, students solve problems when they use mathematics to represent unfamiliar or meaningful situations and plan their approaches.

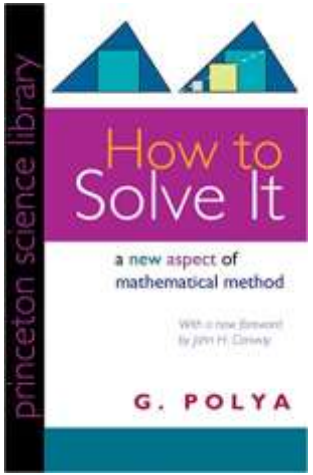
In Years 7–8, students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

In Years 9–10, students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable. Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.

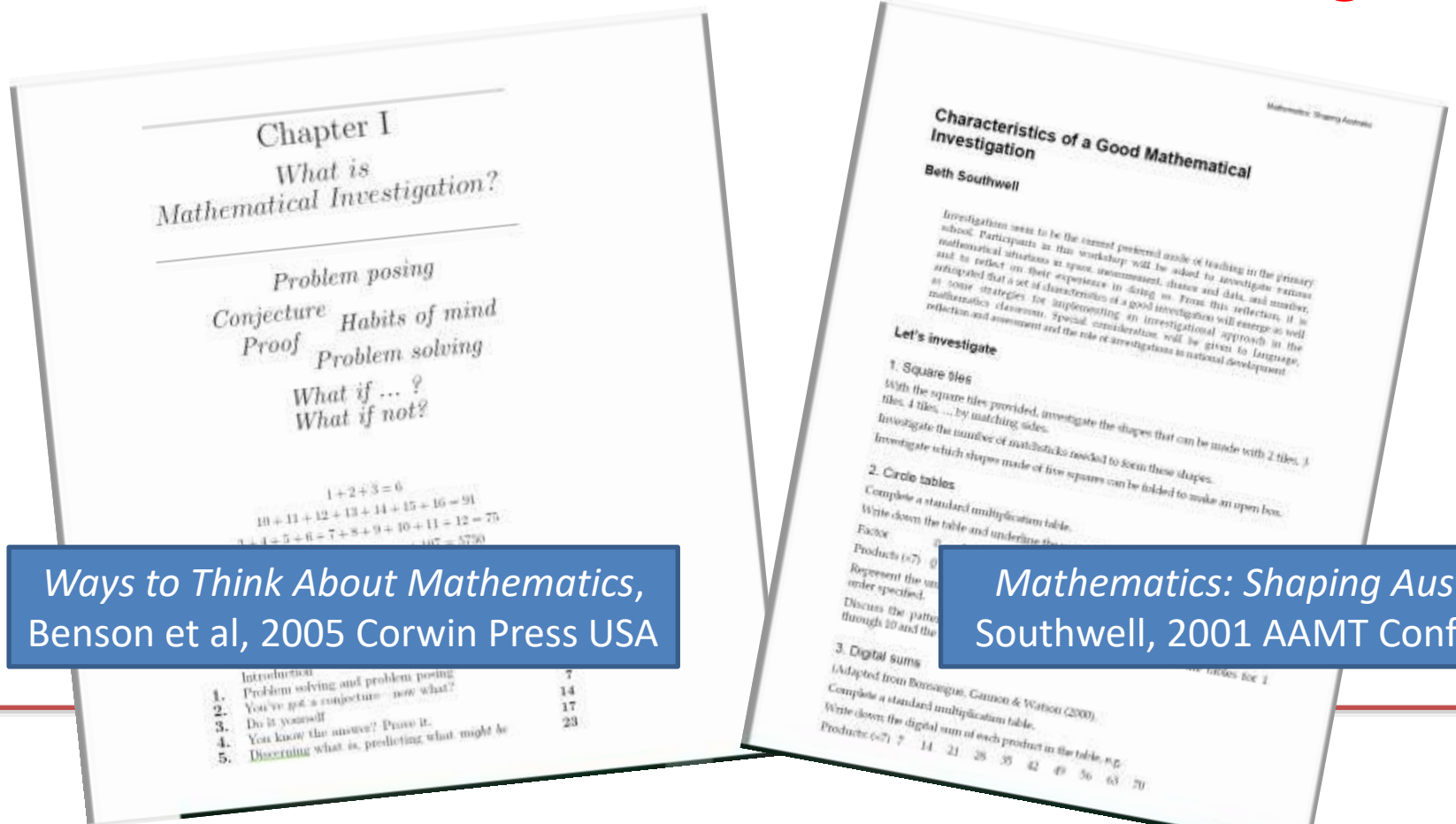
George Polya – 4 Step Process

The 4 stages of problem solving (Polya)

- **Find Out** what the problem is asking for
  - ~ identify relevant data
  - ~ look for the question or verb in the problem
- **Select** a strategy
- **Apply** the strategy
- **Look Back** ~ Does your solution make sense?
  - ~ Reread the original question
  - ~ Should your solution include units?



So what constitutes a Mathematical Investigation?



Chapter I

What is Mathematical Investigation?

Problem posing
Conjecture
Habits of mind
Proof
Problem solving
What if ... ?
What if not?

$1+2+3=6$
$10+11+12+13+14+15+16=91$

Introduction
1. Problem solving and problem posing 7
2. You've got a conjecture – now what? 14
3. Do it yourself 17
4. You know the answer? Prove it. 23
5. Discerning what is, predicting what might be

Characteristics of a Good Mathematical Investigation

Beth Southwell

Let's investigate

1. Square tiles

2. Circle tables

3. Digital sums

Ways to Think About Mathematics, Benson et al, 2005 Corwin Press USA

Mathematics: Shaping Australia, Southwell, 2001 AAMT Conference

Some of the main ideas from these:

- Investigations involve rich problems that invite exploration and the posing of new questions.
- Sometimes called open-ended tasks and this could be taken to imply that investigations cover a range of openness and levels. This suggests that investigations will vary in methodology according to the learners' interest and background.
- When students are asked to investigate, the ideas and difficulties that arise are inevitably less predictable than when the course is all laid out for them in advance. Some problems they pose lead to unanticipated treasures. Others seem likely to take time without giving the students much in return.
- While problem solving is *only* problem solving and has a fairly static feel about it, investigations are problem *posing* and problem *solving* and have a much more dynamic feel. The goal of a problem is given in the problem itself and once the solver has reached an answer (or more than one answer, depending on the problem), the problem is finished. In an investigation, however, the learner sets his or her own goal and the conclusion of the investigation is limited only by the creativity of the learner.

Some of the main ideas from these:

- Investigations involve rich problems that invite exploration and the posing of new questions.
- Sometimes called open-ended tasks and this could be taken to imply that investigations cover a range of openness and levels. This suggests that investigations will vary in methodology according to the learners' interest and background.
- When students are asked to investigate, the ideas and difficulties that arise are inevitably less predictable than when the course is all laid out for them in advance. Some problems they pose lead to unanticipated treasures. Others seem likely to take time without giving the students much in return.
- While problem solving is *only* problem solving and has a fairly static feel about it, investigations are problem *posing* and problem *solving* and have a much more dynamic feel. The goal of a problem is given in the problem itself and once the solver has reached an answer (or more than one answer, depending on the problem), the problem is finished. In an investigation, however, the learner sets his or her own goal and the conclusion of the investigation is limited only by the creativity of the learner.

Some of the main ideas from these:

- One of the fundamental processes used in a mathematical investigation is *conjecturing*. Conjecturing is an essential process in mathematics. It is certainly the process that links mathematics most closely to the scientific method of inquiry from which investigations no doubt derive.
- A most important aspect of the problem-solving process is proof. Often the rationale given for the necessity of proof is that you don't really know that your solution (or conjecture) is correct until you've proven it.
- One of the benefits of a mathematical investigational approach to teaching mathematics is its capability of encouraging students to think and take responsibility for their own learning. It causes students to reflect on what they are doing and learning.
- Presenting a mathematical investigation can also be extremely valuable to your students. It is one contributor to the development of their ability to communicate.

Some of the main ideas from these:

- One of the fundamental processes used in a mathematical investigation is *conjecturing*. Conjecturing is an essential process in mathematics. It is certainly the process that links mathematics most closely to the scientific method of inquiry from which investigations no doubt derive.
- A most important aspect of the problem-solving process is proof. Often the rationale given for the necessity of proof is that you don't really know that your solution (or conjecture) is correct until you've proven it.
- One of the benefits of a mathematical investigational approach to teaching mathematics is its capability of encouraging students to think and take responsibility for their own learning. It causes students to reflect on what they are doing and learning.
- Presenting a mathematical investigation can also be extremely valuable to your students. It is one contributor to the development of their ability to communicate.

Activities

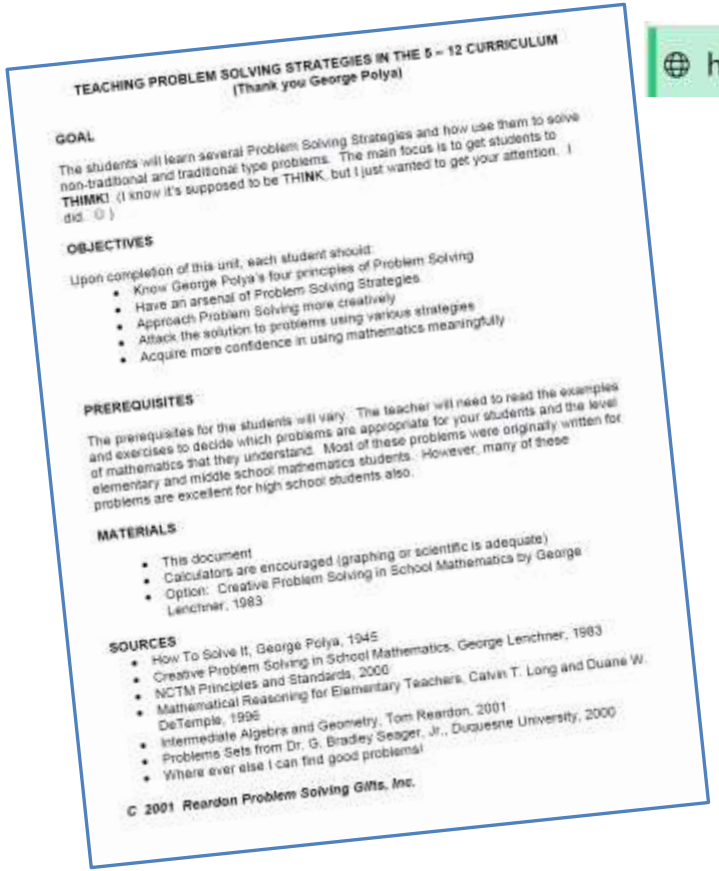
- Crazy Animals
- Handshakes
- Birth Month Match
- Left/Right Game

highlighted pedagogy

- Computational Thinking
- Problem Solving
- Simulation
- Investigation

Resources for Computational Thinking

Problem Solving unit from Tom Reardon



TEACHING PROBLEM SOLVING STRATEGIES IN THE 5 – 12 CURRICULUM
(Thank you George Polya)

GOAL

The students will learn several Problem Solving Strategies and how use them to solve non-traditional and traditional type problems. The main focus is to get students to **THINK**! (I know it's supposed to be THINK, but I just wanted to get your attention. I did. ☺)

OBJECTIVES

Upon completion of this unit, each student should:

- Know George Polya's four principles of Problem Solving
- Have an arsenal of Problem Solving Strategies
- Approach Problem Solving more creatively
- Attack the solution to problems using various strategies
- Acquire more confidence in using mathematics meaningfully

PREREQUISITES

The prerequisites for the students will vary. The teacher will need to read the examples and exercises to decide which problems are appropriate for your students and the level of mathematics that they understand. Most of these problems were originally written for elementary and middle school mathematics students. However, many of these problems are excellent for high school students also.

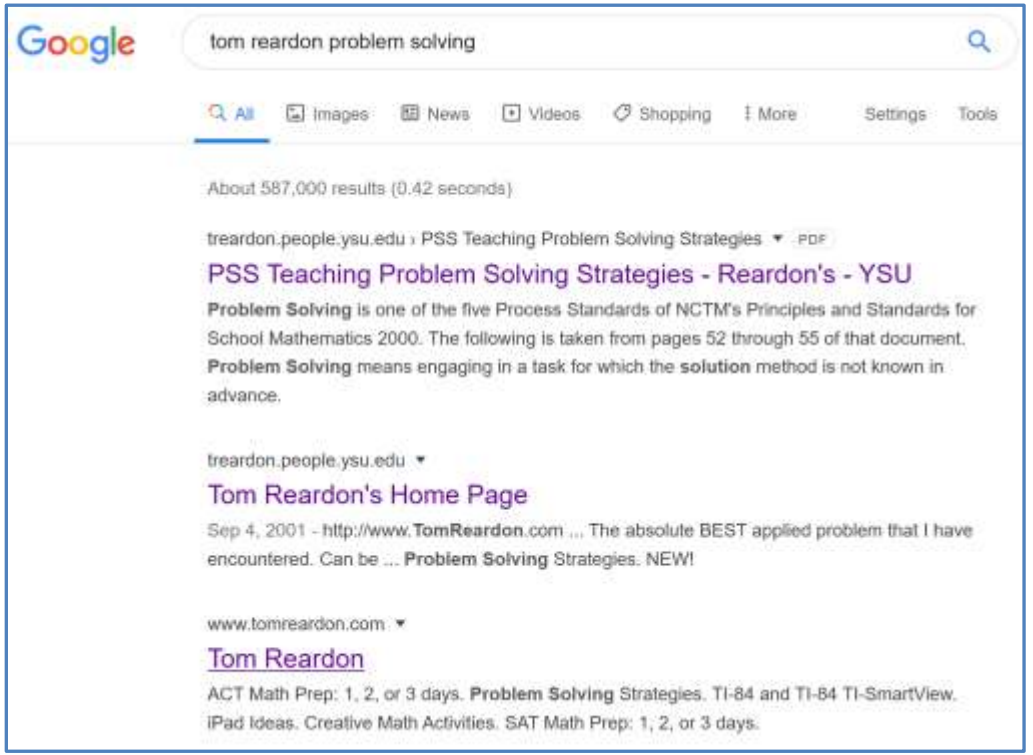
MATERIALS

- This document
- Calculators are encouraged (graphing or scientific is adequate)
- Option: Creative Problem Solving in School Mathematics by George Lenchner, 1983

SOURCES

- How To Solve It, George Polya, 1945
- Creative Problem Solving in School Mathematics, George Lenchner, 1983
- NCTM Principles and Standards, 2000
- Mathematical Reasoning for Elementary Teachers, Calvin T. Long and Duane W. DeTemple, 1996
- Intermediate Algebra and Geometry, Tom Reardon, 2001
- Problems Sets from Dr. G. Bradley Seager, Jr., Duquesne University, 2000
- Where ever else I can find good problems!

http://**treardon.people.ysu.edu/PSS Teaching Problem Solving Strategies.pdf**



Google tom reardon problem solving

All Images News Videos Shopping More Settings Tools

About 587,000 results (0.42 seconds)

treardon.people.ysu.edu › PSS Teaching Problem Solving Strategies ▾ PDF

PSS Teaching Problem Solving Strategies - Reardon's - YSU

Problem Solving is one of the five Process Standards of NCTM's Principles and Standards for School Mathematics 2000. The following is taken from pages 52 through 55 of that document. **Problem Solving** means engaging in a task for which the **solution** method is not known in advance.

treardon.people.ysu.edu ▾

Tom Reardon's Home Page

Sep 4, 2001 - http://www.**TomReardon**.com ... The absolute BEST applied problem that I have encountered. Can be ... **Problem Solving** Strategies. NEW!

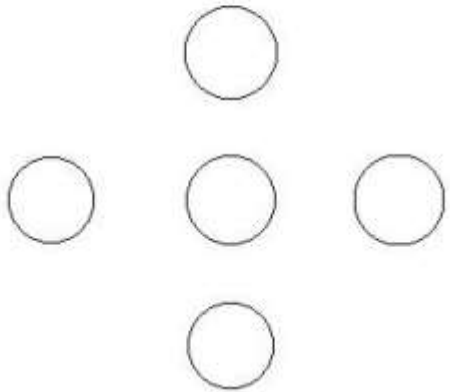
www.tomreardon.com ▾

Tom Reardon

ACT Math Prep: 1, 2, or 3 days. **Problem Solving** Strategies. TI-84 and TI-84 TI-SmartView. iPad Ideas. Creative Math Activities. SAT Math Prep: 1, 2, or 3 days.

PSS 1 ***GUESS AND CHECK***

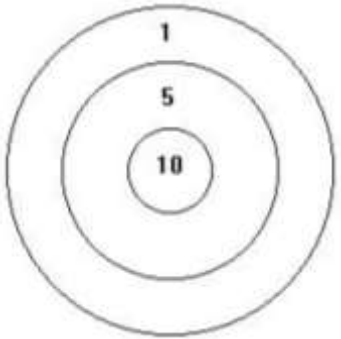
EX. 1 Copy the figure below and place the digits 1, 2, 3, 4, and 5 in these circles so that the sums across (horizontally) and down (vertically) are the same. Is there more than one solution?



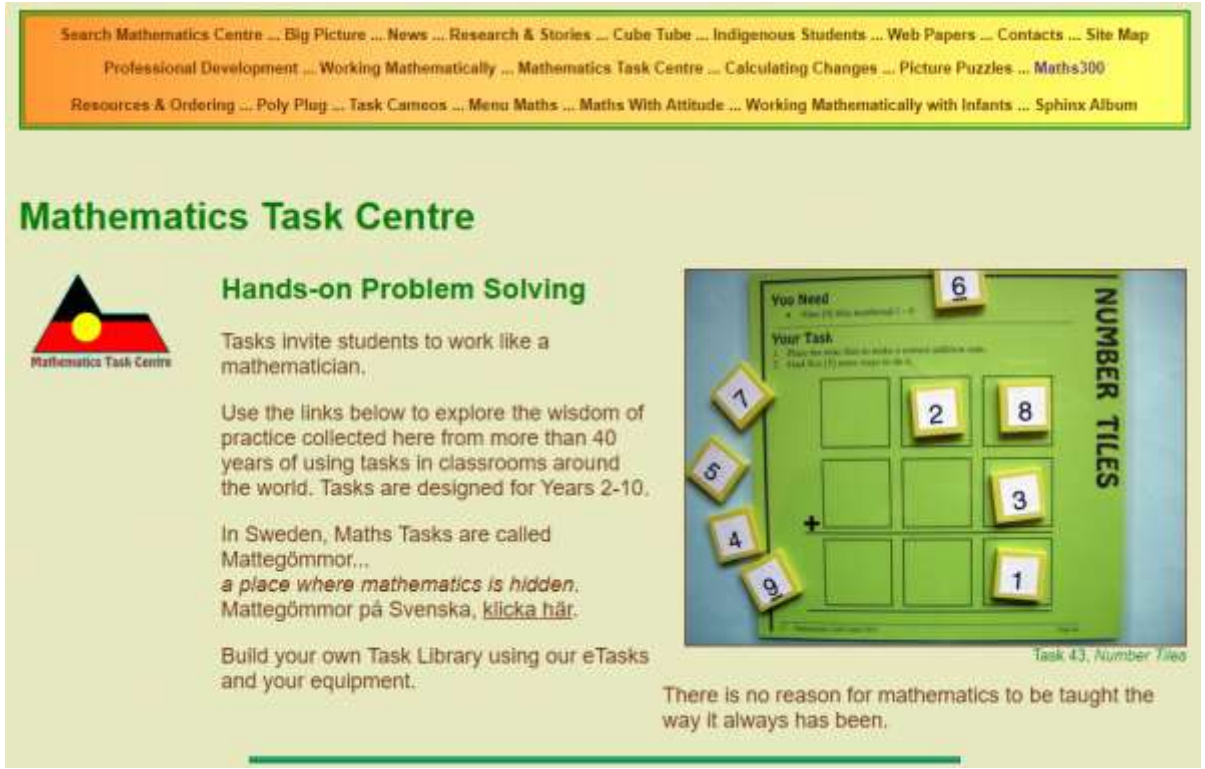
DAY 2 ***PSS 2*** ***MAKE AN ORGANIZED LIST***

EX. 2

Three darts hit this dart board and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1's, two 1's and 5, one 5 and two 10's, And so on. How many different possible total scores could a person get with three darts?

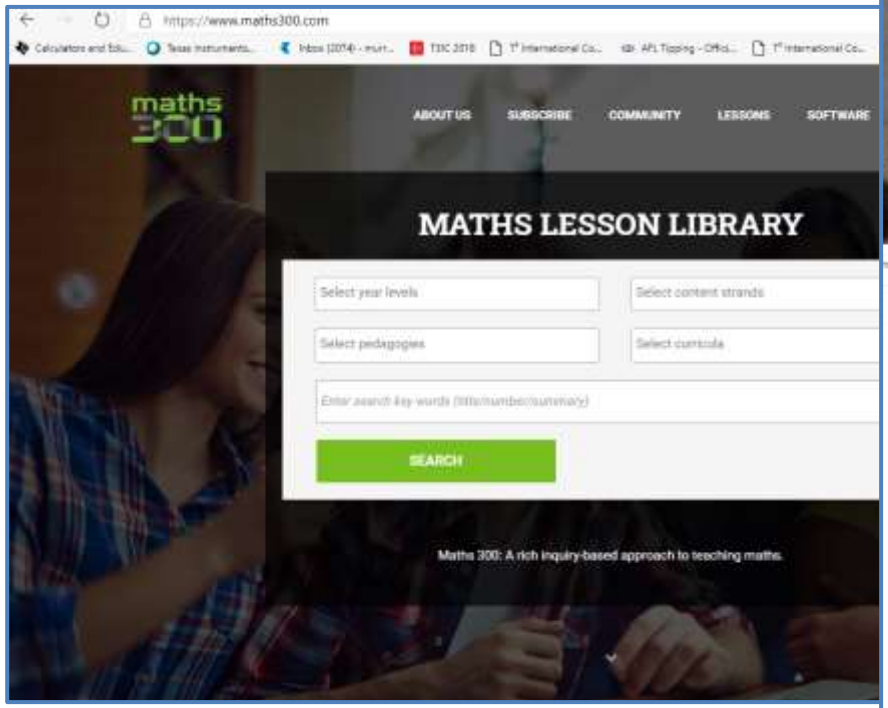


Resources from Mathematics Task Centre



http://www.mathematicscentre.com/taskcentre/

Resources from Maths 300



https://www.maths300.com/

Resources from Texas Instruments

TEXAS INSTRUMENTS | Education Technology | Search

Products | Teachers | Students | Support | Downloads | Where to Buy | Site: Australia and New Zealand

- Australian Curriculum Nspired
- Year 7
- Year 8
- Year 9
  - Measurement and Geometry
  - Number and Algebra
  - Statistics and Probability
  - STEM
- Year 10
- Year 10A

Year 9: Paving Problem

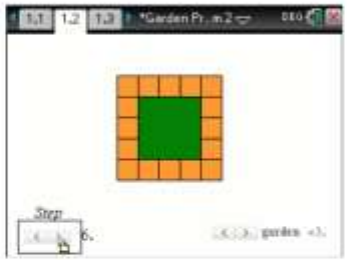
by Texas Instruments

Objectives

- **ACMNA213**: Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Vocabulary

- Factorise
- Expand
- Simplify



About the Lesson

A square garden bed is surrounded by pavers, how many pavers are required for a 1m x 1m garden? What about a 2m x 2m or n x n garden. Students use animations that support the development of the rules followed by algebraic techniques such as factorise, expand and simply to illustrate that these abstract algebraic representations are equivalent. Students take this knowledge and experience and apply it to a different style of garden bed.

Download
Download All Files (ZIP)
TInspire Activity File 1
TInspire Activity File 2
TInspire Activity File 3
TInspire Activity File 4
Student Activity
Teacher Notes and Answers

Device
TI-Nspire™ CX
TI-Nspire™ CX CAS

Software
TI-Nspire™
TI-Nspire™ CAS

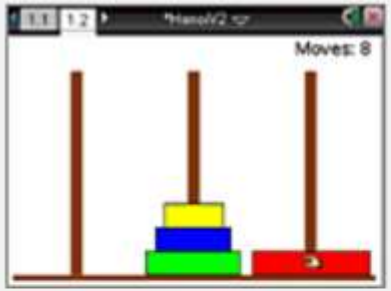
Standards
ACMNA213

TI-Nspire Version
4.5

Report an Issue

TEXAS INSTRUMENTS

Resources from Texas Instruments



Standard ACMNA296

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations

Activity: Towering Mathematics

The classic "Towe
discs of different
restrictions that la
discs and discs c
challenge is to m
another. This acti
problem and to fi
of moves and the
contains a virtual

Standard ACMNA296

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations

Activity: Leap Frog

This classic puzzle requires all the frogs on the left side of the pond to reach the right side and vice versa. How many moves does it take to solve the puzzle? What if there were more frogs? Click on the frogs in this TI-Nspire document to make them move, the moves are counted automatically. Set the number of frogs on each side and start solving!

Resources for Investigations:

- MAV SACS - Suggested Starting Points

  www.mav.vic.edu.au/Resources/VCE-Resources
- VCAA Support Material

  VCE Mathematics School-assessed Coursework (SAC) Resource (2000)

  - Advice For Teachers – Assessment Tasks, Resources
- Assessment task exemplars - Mathematical Methods – SACE
- Chance & Data Investigations, Lovitt & Lowe, 1993
- www.maths300.com
- www.itute.com free maths assessment tasks
- www.Plus.maths.org
- International Mathematical Modelling Challenge
- www.education.ti.com/en-au

Just a few more interesting links:

https://www.mathsisfun.com/data/quincunx.html

https://www.mathsisfun.com/data/quincunx-explained.html

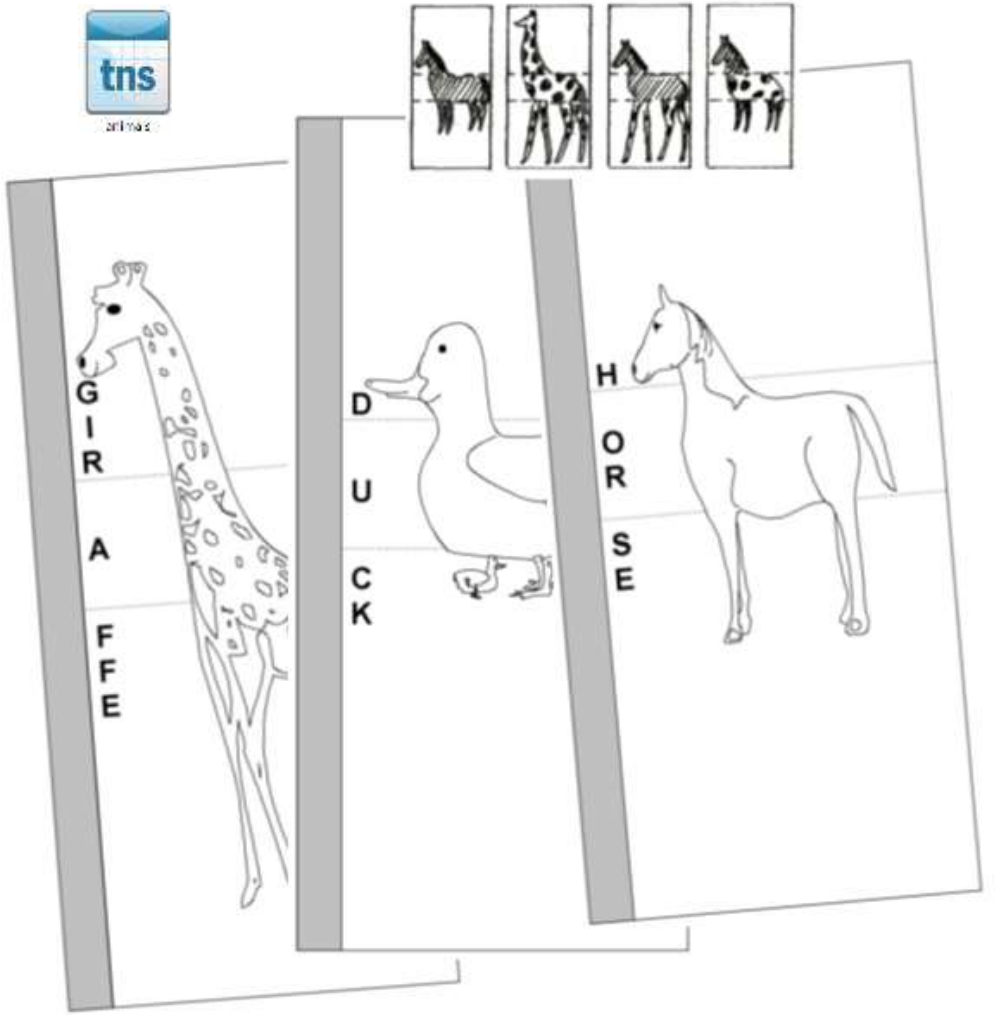
https://galtonboard.com/

Crazy Animals

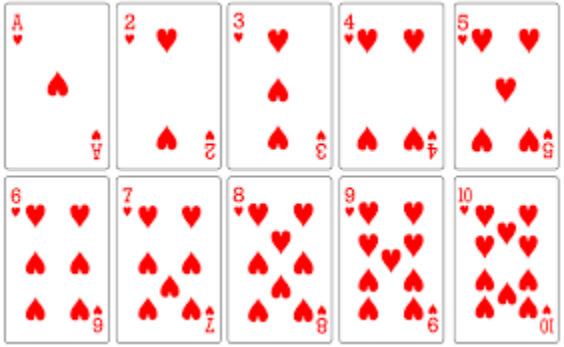
Computational Thinking

How many different animals can you make with the picture book?

What are their names?



A challenge for you:



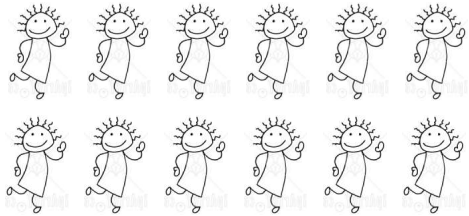
You have a set of cards numbered 1 to 10.
Shuffle the cards into a random order.
Now design an algorithmic process by which you could instruct another person (or a robot) to place the cards in order from lowest to highest.
Is your algorithm absolutely foolproof?
Could your algorithm be represented on a flowchart or coded for a computer to execute?

Handshakes

Problem Solving

- Polya 4 steps
- Handshake problem

If there are 12 people at a party and each person shakes hands with each other person, then how many handshakes are there in total?



- The richness of moving from closed questions to **open problems**

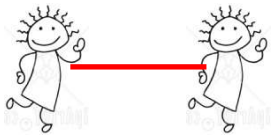
Four Steps

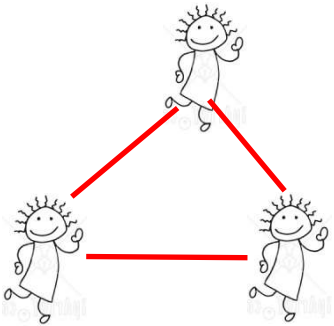
- Find Out
- Select Strategy
- Apply Strategy
- Look Back

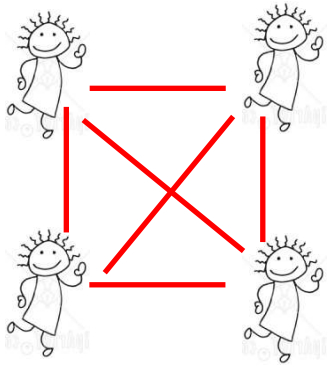
Strategies list

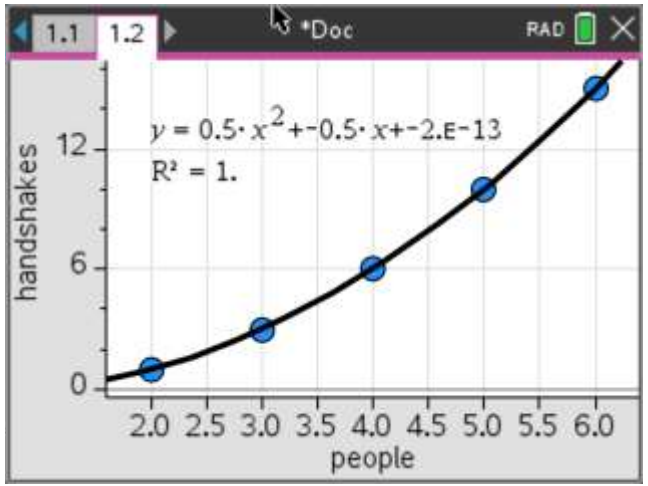
- Experiment
- Role play
- Diagram
- Make an organised list
- Solve a simpler problem
- Look for a pattern
- Develop a formula
- Work backwards
- Construct a table
- Sketch a graph
- …….

Strategies List – The handshake problem







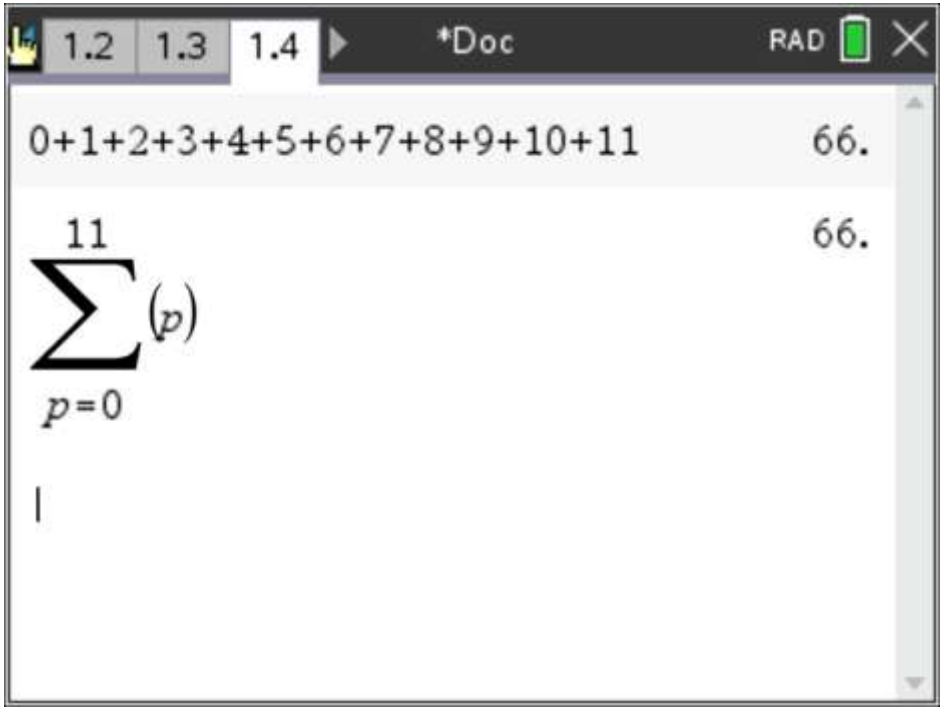


people	handshakes
1	
2	1
3	3
4	6
5	10
12	?

+2 +3 +4

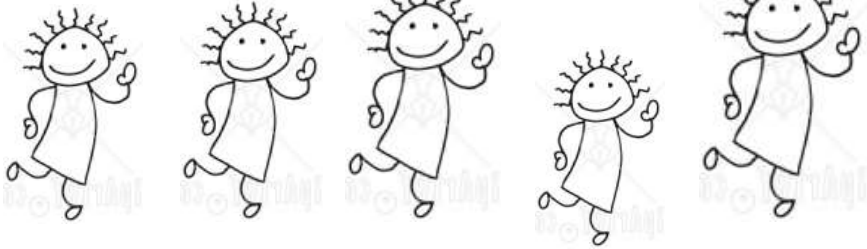
$$h(12) = \frac{12(12-1)}{2} = 66$$

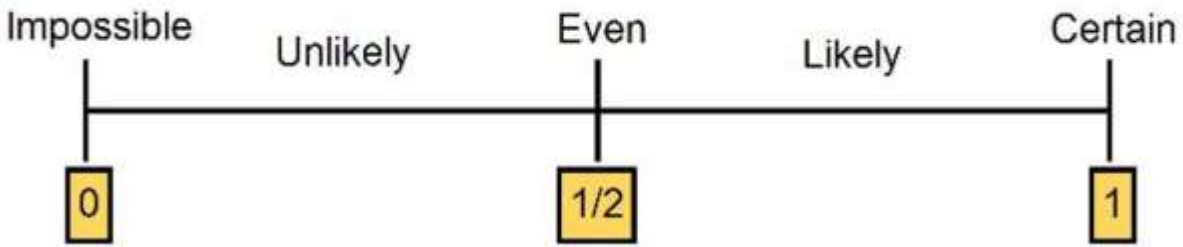
Strategies List – The handshake problem



Birth Month Match

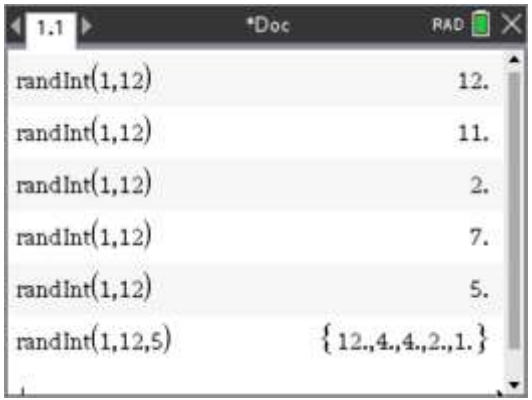
In a randomly selected group of 5 people, what is the probability that there will be at least one match in birth months?





Birth Month Match

- Collect data from real samples
- Simulate data (e.g. by drawing number cards, dodecahedral dice, table of random numbers)
- Collect data from electronic simulation



Further Investigation:

- **What if the group was bigger or smaller?**
- **Is there a relationship between sample size and probability of a match?**
- **What is the likelihood of a birth *week* match, or a birth *day* match?**

Theoretical solution

Pr (match) = 1 – Pr (no match)

$$= 1 - \left(\frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12}\right)$$

$$= 1 - \frac{95040}{248832}$$

$$= 1 - \frac{55}{144}$$

$$= 0.6181$$

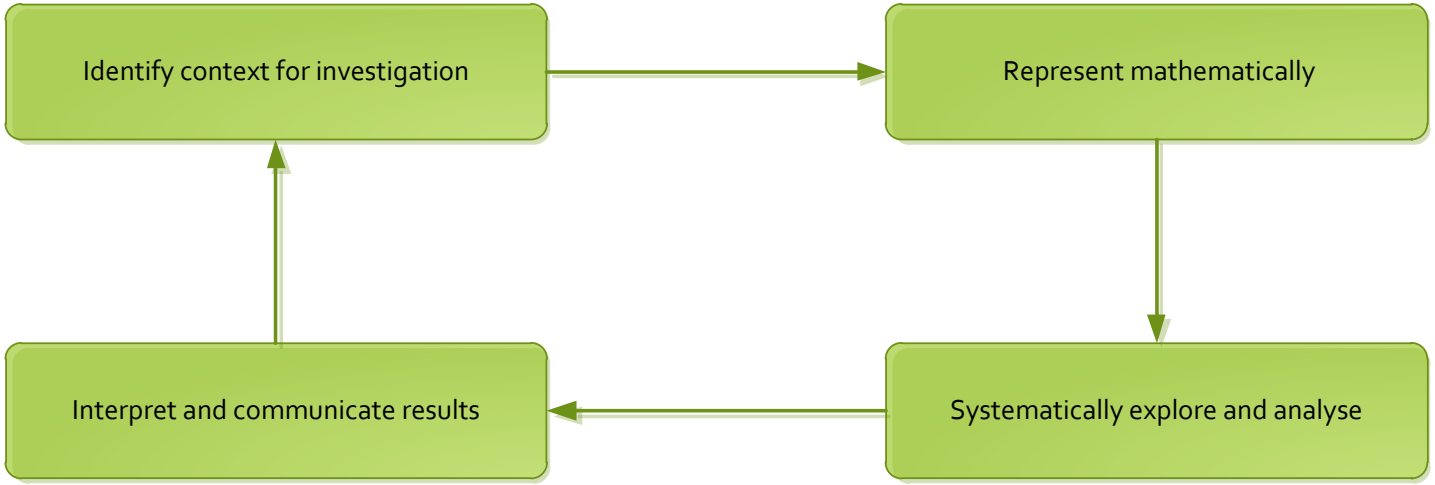
Mathematical investigation

Is the process of using mathematical constructs, structures, concepts, processes and skills to represent and explore aspects of a situation or context in a way that enables one to investigate characteristics, features and behaviour of systems and objects related to the context.

This includes the use of data, sets, dependent and independent variables, constants and parameters, relations and functions, graphs, tables, charts and diagrams, equations and inequalities, variation, generalization, specialisation, experiment, computation and proof.

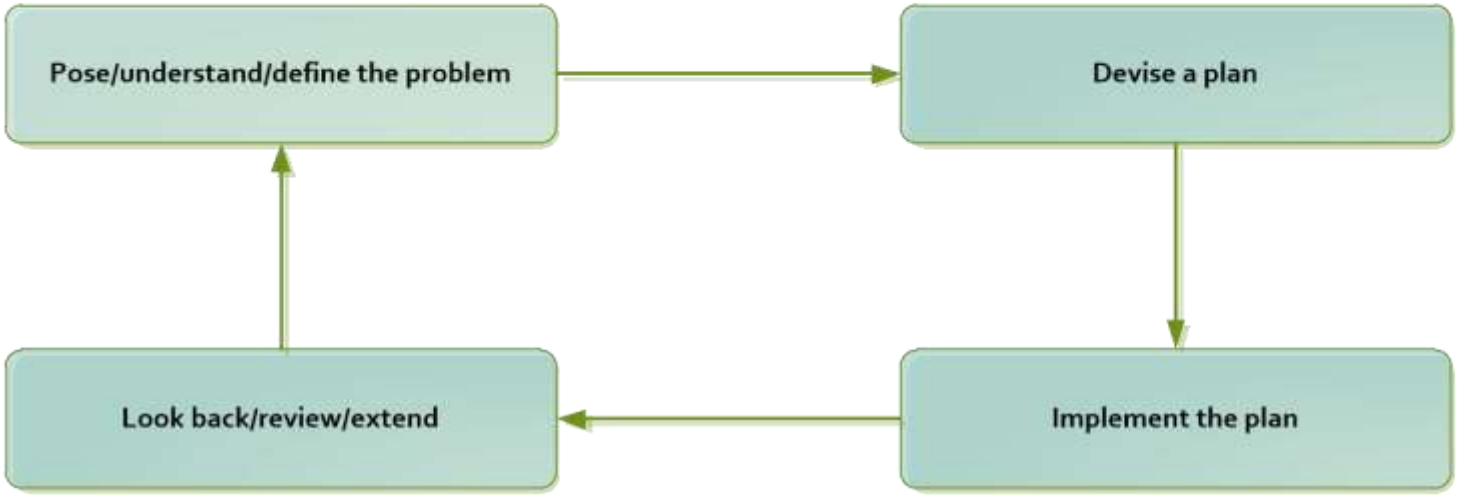
Mathematical investigation schema

A simple representation for the mathematical investigation process is shown below:



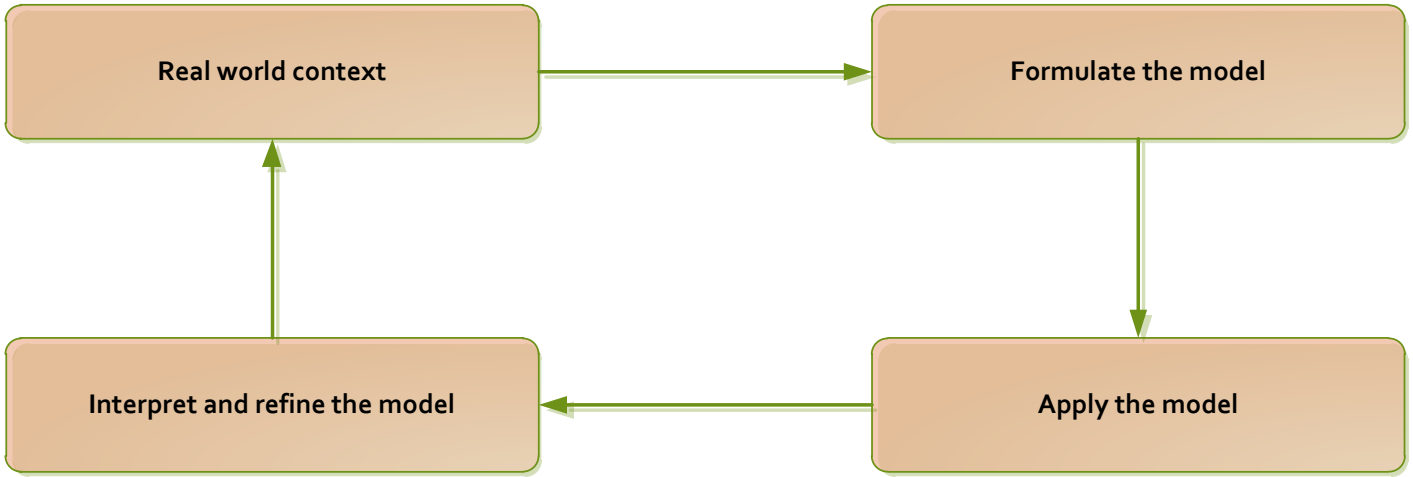
Problem-solving schema

A simple and well known diagrammatic representation of the problem-solving process, adapted from *How to Solve It* (Polya, 1945, Princeton University Press) follows.

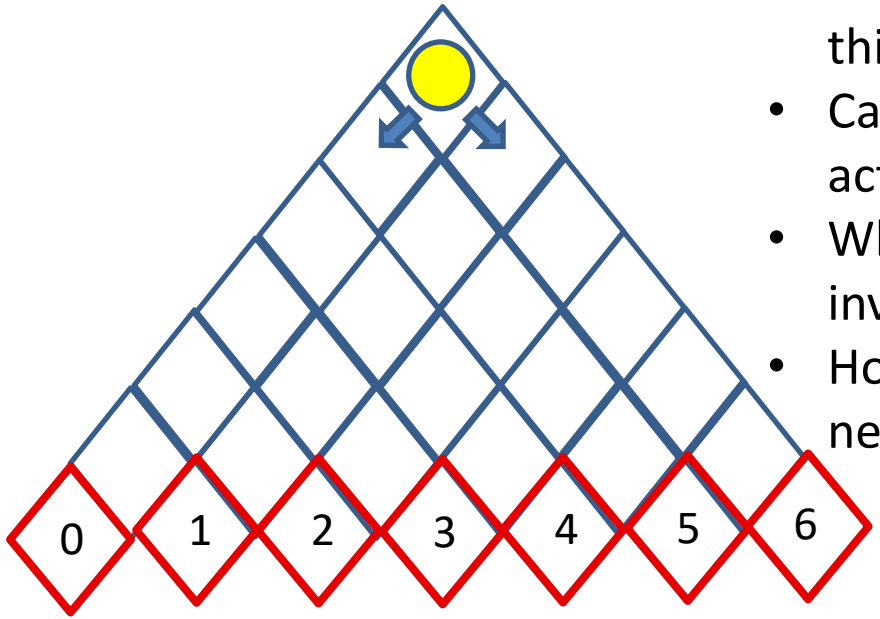


Modelling schema

A simple and well known diagrammatic representation of the mathematical modelling process is shown below.

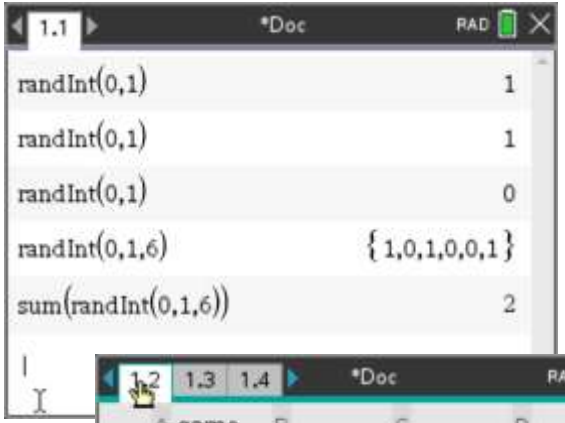


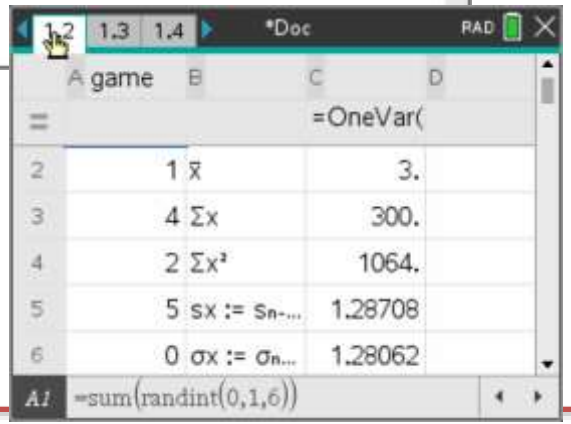
An investigation for you to start on:

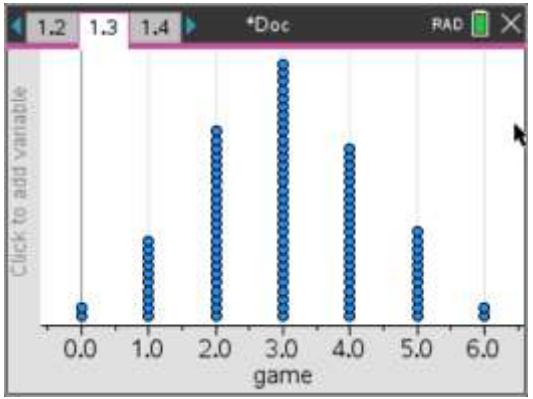


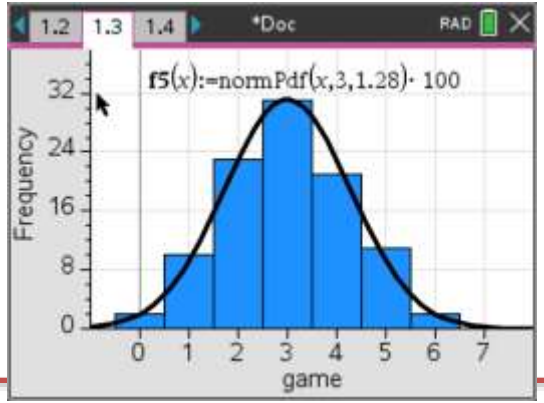
- What mathematical questions can arise from this situation?
- Can you identify possible links between this activity and the current study design?
- What extensions to this activity might a student investigate?
- How might this situation be modified such that new questions and problems may be posed?

Simulation & Analysis with technology:









Results from a simulator:

[Quincunx (Galton Board) (mathsisfun.com)]